Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 1

The main topics for this problem set are representation of signals and systems in the time domain and sampling. Relevant chapters from the textbook are 1.3.1, 1.3.2, 1.4.1, 2.1, 2.2, and 2.3. You will need headsets in order to do Problem 2. The maximum score for each problem is given in parentheses.

Problem 1 (2 points)

Two signals $x[n]$ and $y[n]$ are given by

$$x[n] = \begin{cases} 5-n & 0 \leq n \leq 4 \\ 0 & \text{otherwise,} \end{cases} \qquad \text{and} \qquad y[n] = \begin{cases} 1 & 2 \leq n \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch $x[n]$ and $y[n]$.

(b) Sketch $x[n-k]$ for $k = 3$ and $k = -3$.

(c) Sketch $x[-n]$.

(d) Sketch $x[5-n]$.

(e) Sketch $x[n] \cdot y[n]$.

(f) Express the signal $x[n]$ by using the unit sample sequence $\delta[n]$.

(g) Express the signal $y[n]$ by using the unit step signal $u[n]$.

(h) Compute the energy of the signal $x[n]$.

Problem 2 (2 points)

A discrete harmonic sequence $x[n]$ with normalized frequency f_1 is given, i.e

$$x[n] = A\cos(2\pi f_1 n) \tag{1}$$

Further assume that the sequence is generated by sampling an analogue signal $x(t)$ with rate F_s.

(a) Which physical frequencies F_1 can f_1 correspond to if $F_s = 6000\text{Hz}$?

(b) Use Matlab or Python to generate a sequence of length 4 seconds of $x[n]$.

(c) Use the Matlab command `soundsc` or the Python command `sounddevice.play` to listen to the harmonic when the normalized frequency $f_1 = 0.3$ and the sampling rate F_s is given by respectively 1000Hz, 3000Hz and 12000Hz.
Comment on what you hear.

(d) Now assume a fixed sampling rate $F_s = 8000\text{Hz}$ while the physical frequency F_1 is respectively 1000Hz, 3000Hz and 6000Hz.
Comment on what you hear.
Relate it to the corresponding normalized frequency f_1.

Problem 3 (2 points)

Several discrete-time systems are given below. State whether each system is linear, time-invariant, and causal. Justify your answers.

(a) $y[n] = x[n] - x^2[n-1]$

(b) $y[n] = nx[n] + 2x[n-2]$

(c) $y[n] = x[n] - x[n-1]$

(d) $y[n] = x[n] + 3x[n+4]$

Problem 4 (2 points)

Two causal systems are given by the following difference equations

$$y[n] = x[n] + 2x[n-1] + x[n-2] \tag{2}$$
$$y[n] = -0.9y[n-1] + x[n]. \tag{3}$$

(a) Find the unit pulse responses $h_1[n]$ and $h_2[n]$ of the systems.

(b) State whether each system is FIR or IIR. Justify your answer.

(c) Determine whether each system is stable.

(d) Represent the filters graphically by a filter structure.

Problem 5 (2 points)

The signal

$$x[n] = \begin{cases} n+1 & 0 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

is passed through two systems as shown in Figure 1.

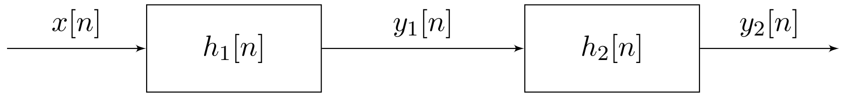

Figure 1: Filtering with two systems

The unit sample responses of the systems are given by

$$h_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$
$$h_2[n] = \begin{cases} 0.9^n & 0 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute and sketch the signal $y_1[n]$.

(b) Use Matlab or Python to compute and plot the signal $y_2[n]$. Useful Matlab function are `conv` and `stem`. Useful Python functions are `sounddevice.play` and `plt.steam`.

(c) What is the relationship between signal lengths at the input and the output of the two systems?

(d) Use Matlab or Python to compute and plot the signals $y_1[n]$ and $y_2[n]$ when the order of the two systems in Figure 1 is interchanged. Compare with the plots in (a) and (b) and comment.