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Data flow analysis instances

Where we were

- We have gone through Live Variables
  - By intuition
  - As a set of constraint equations that converge to a fixed point when you solve them iteratively
  - Looking at how its solutions correspond to positions in a lattice

  and argued that the general method works for different types of information as well.
- Today, we'll try it out with
  - A few different types of elements in the sets
  - Different constraints

The framework ingredients

- A *domain* of things to analyze
  - Sets of variables (Liveness)
  - Sets of copies
  - Sets of expressions
  - Sets of variable definitions…
- A *transfer function*
  - Gives a forward/backward direction
  - Says how to change the sets based on the program logic
- A *meet operator*
  - Says what to do when control flow paths collide

Copy propagation

- Some variables can be copies of each other, let us detect them
  - Liveness is a backward analysis that adds set elements from any path to a program point
  - Copy propagation is a forward analysis that restricts set elements to those that are valid along every path to a program point
- We can work copy propagation out by intuition as well, to illustrate the effects of direction and choice of meet operator

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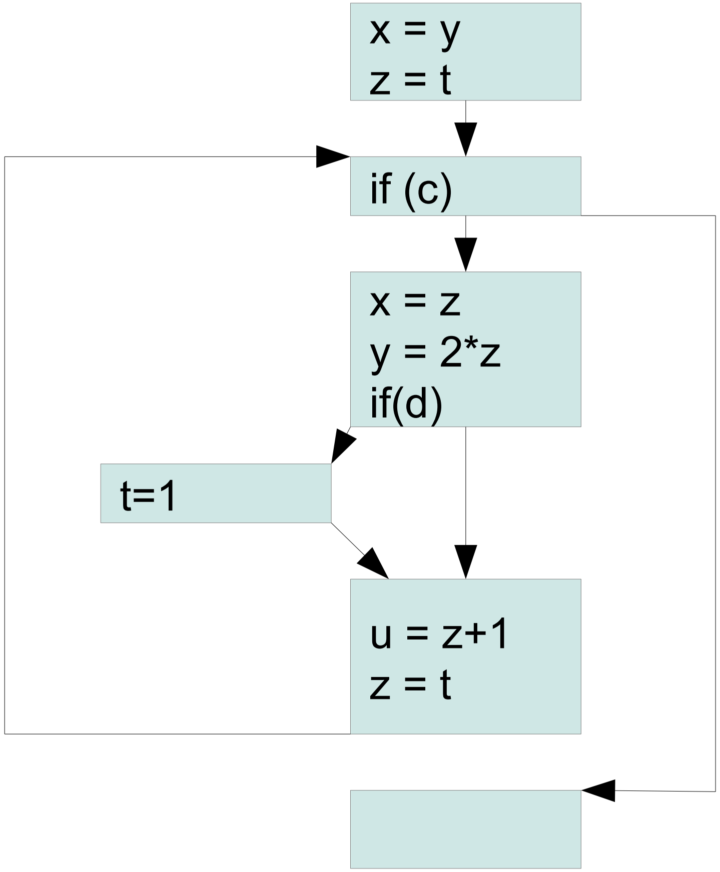
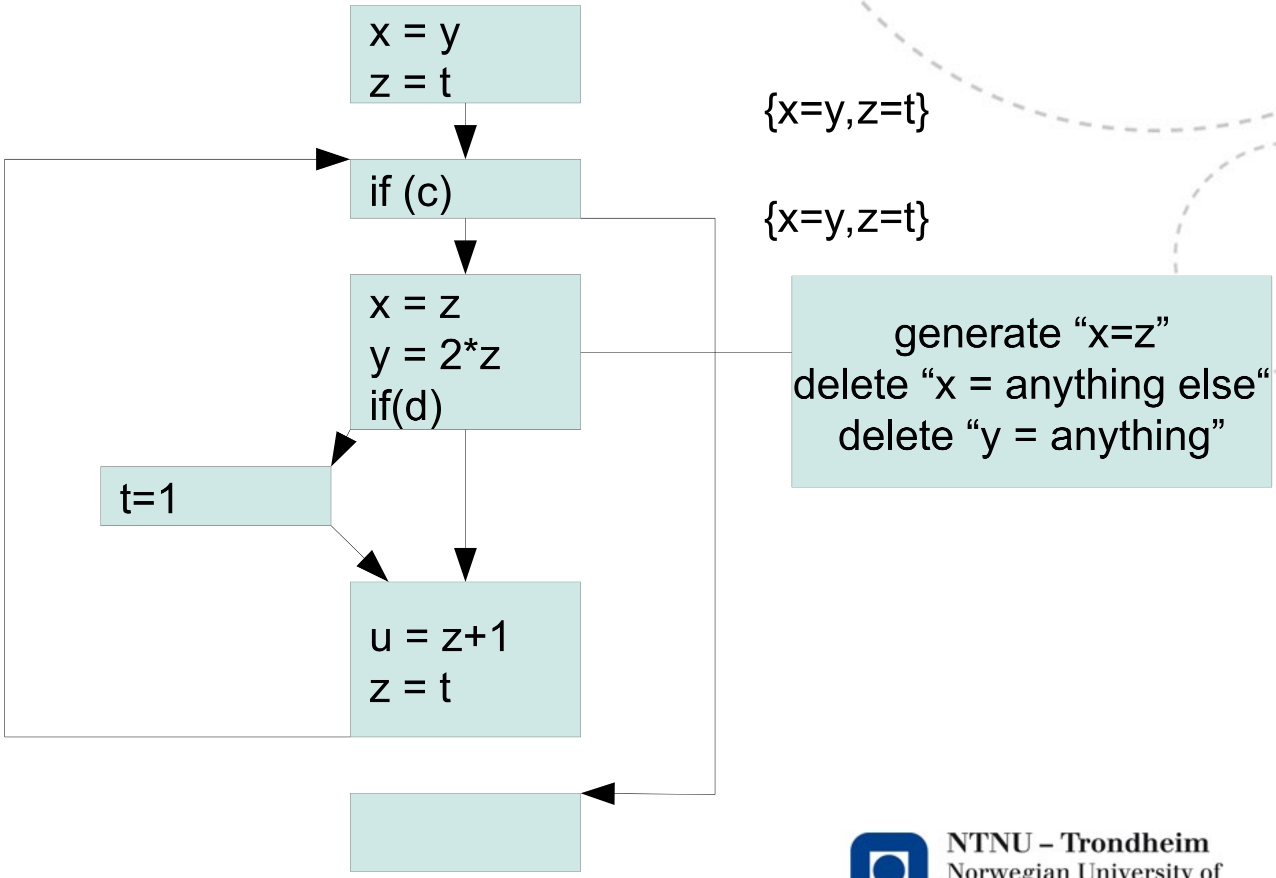
Copy propagation

- I've modified the running example a bit, so that there are some copies to detect



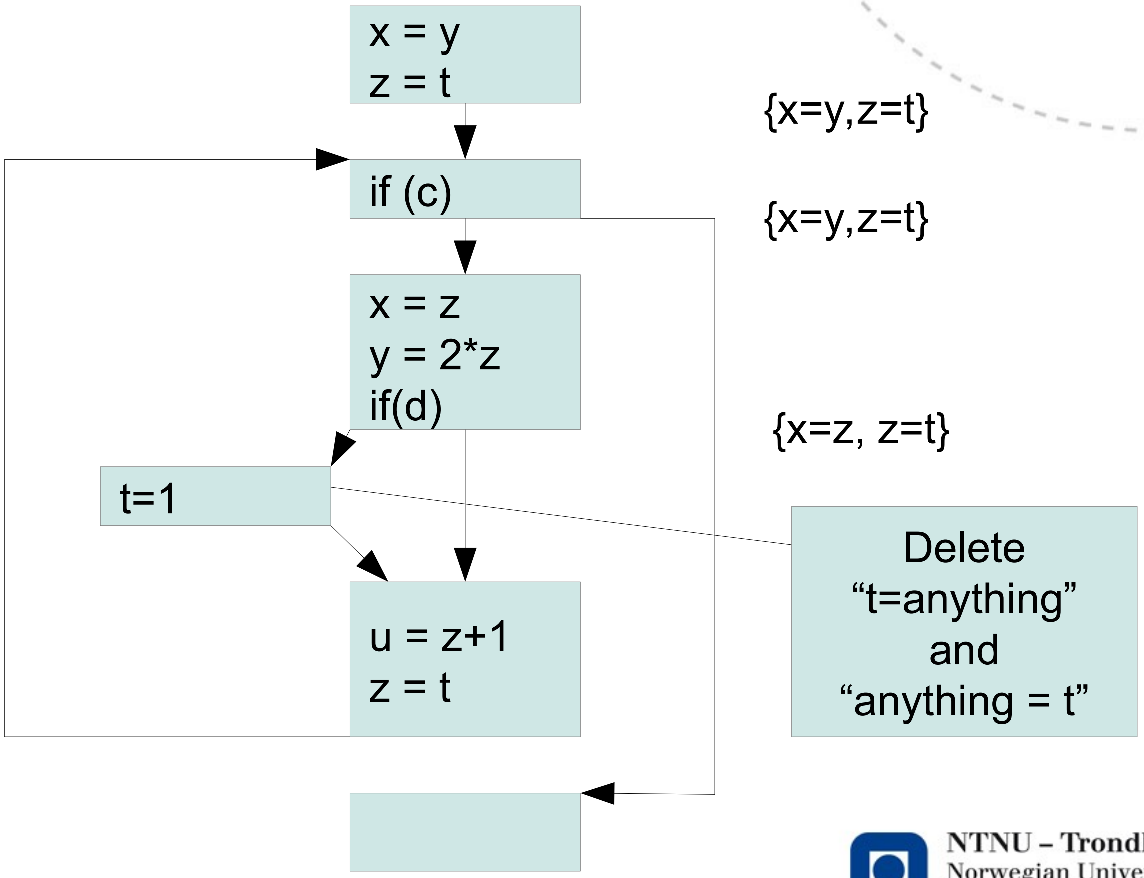
Copy propagation

- The first two statements create some copies to start with
- The loop body makes x a copy of another variable than it was before



Copy propagation

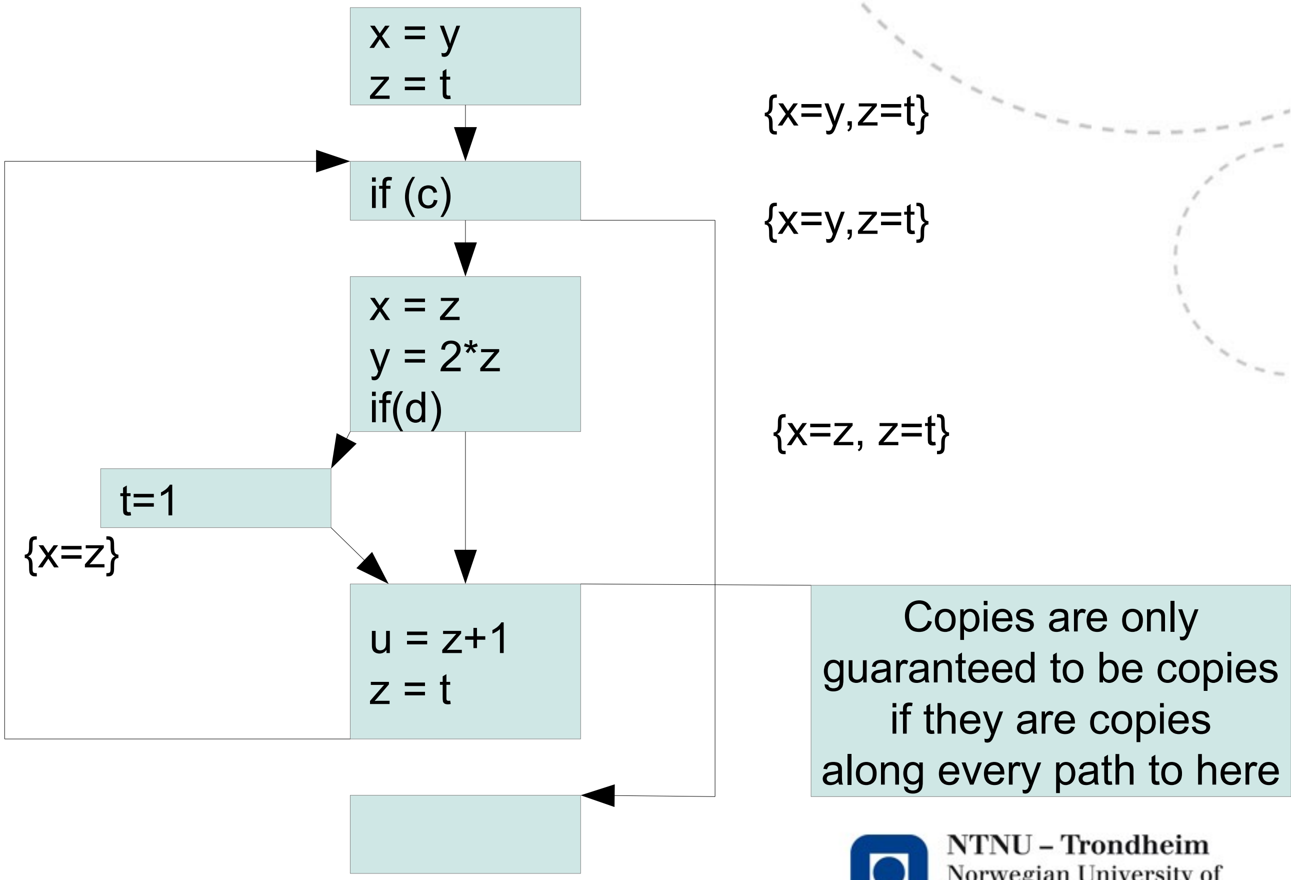
- Here's an assignment which stops t from being a copy of any other variable



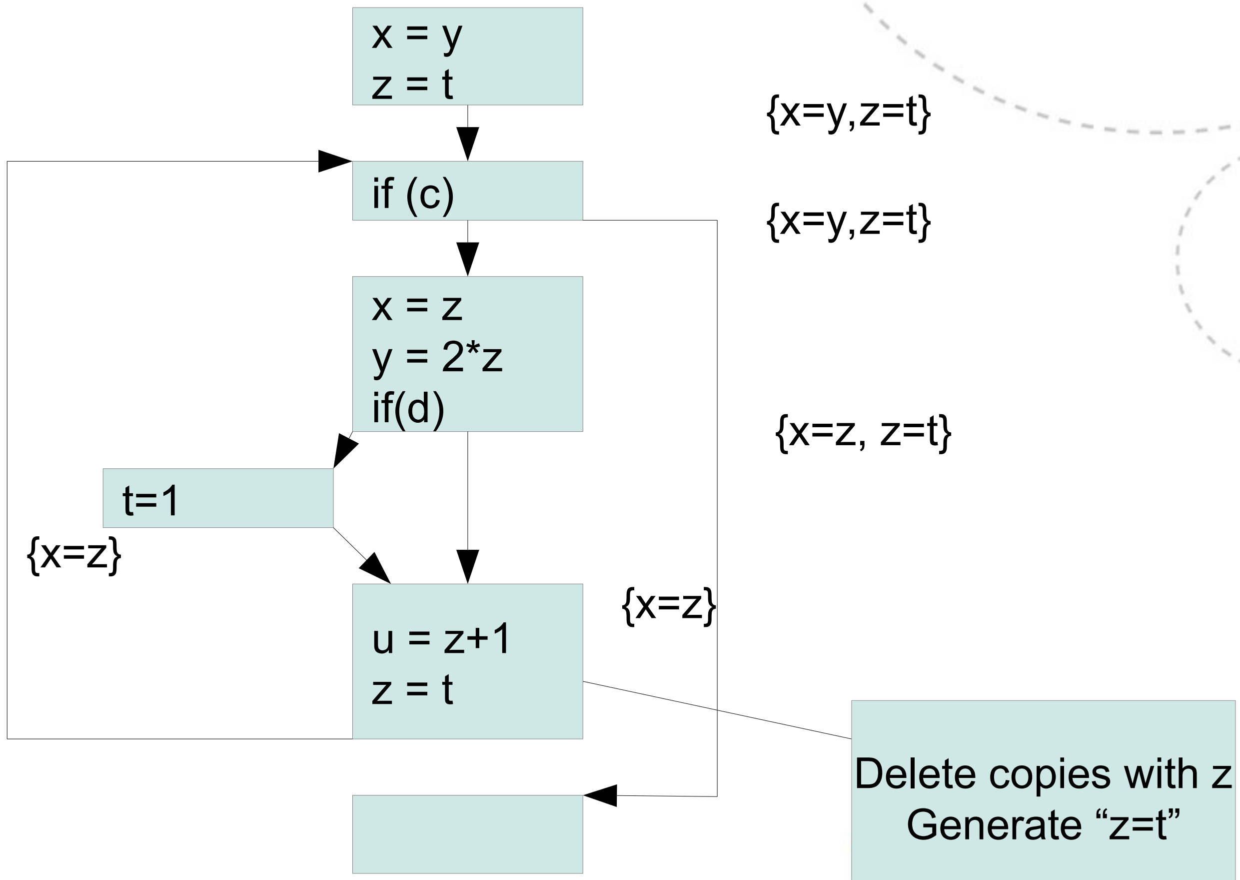
Copy propagation

- Control flows meet:

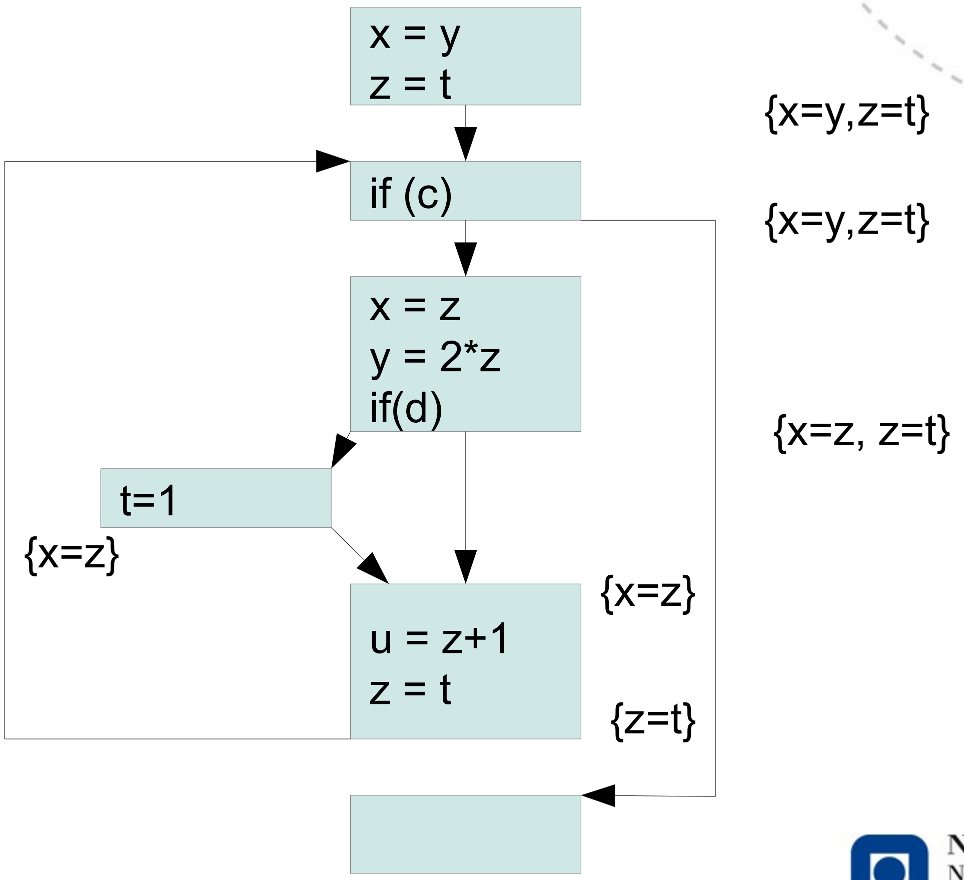
  To be sure that we can treat two variables as copies of each other, there can't be any possibility that they're different



Copy propagation



Copy propagation

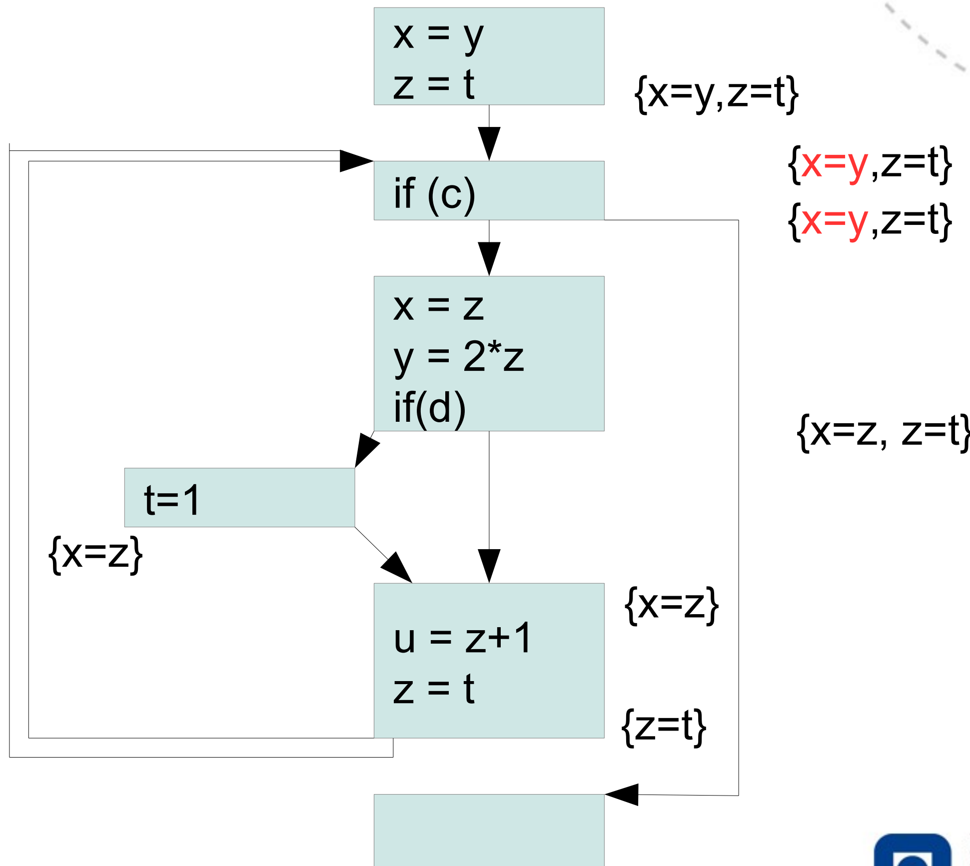


Iteration 2

- We've found a path to the loop head where x=y is not necessarily true
- Take it away:

  {x=y,z=t} ⋂ {z=t}

  = {z=t}



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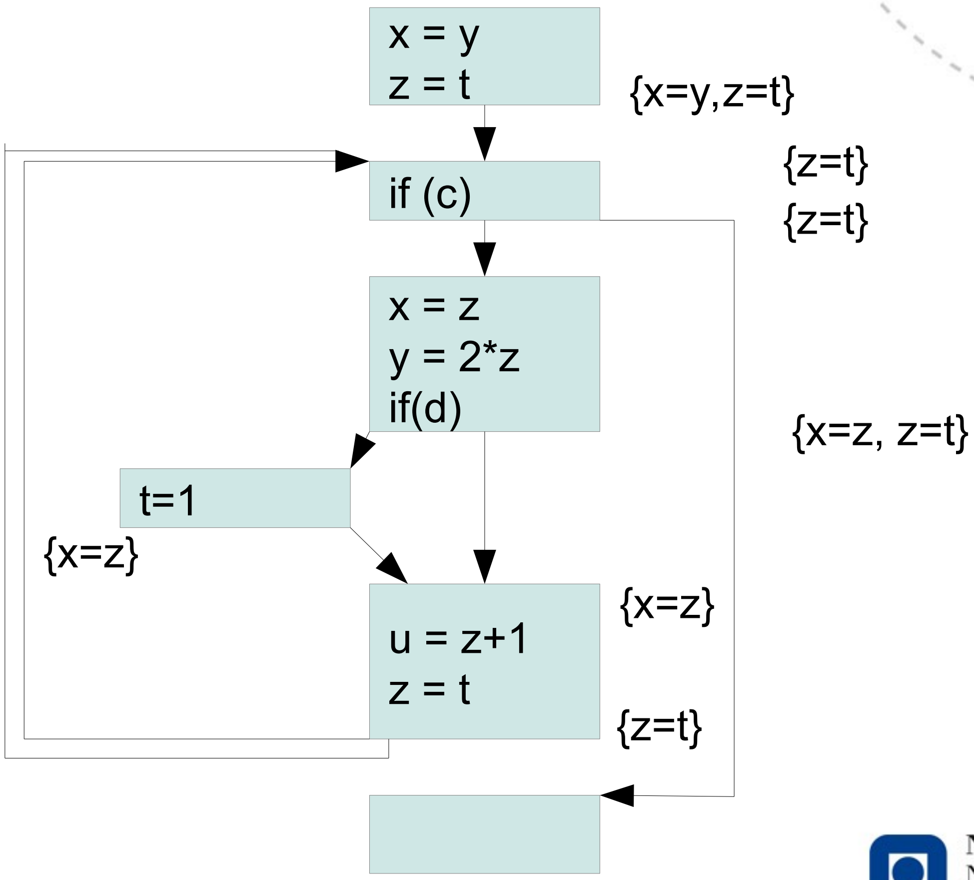
Solution



Available Expressions

- An *available expression* is an expression evaluated in all program executions, which would have the same value if re-evaluated
- The sets we look for are sets of expressions, so we need to number all of those

Available Expressions Data flow equations

- Instructions:

  $out[B] = F_B(in[B])$ (Forward analysis)

- Control flow:

  $in[B] = \bigcap \{ out[B'] \mid B' \in pred(B) \}$ (Meet op. is intersection)

- Interpretation:

  An expression is available at the entry of B if it is available at the exit of all predecessor blocks

Available Expressions Transfer function

$F_l(X) = \{ X - kill [l] \} \cup gen [l]$

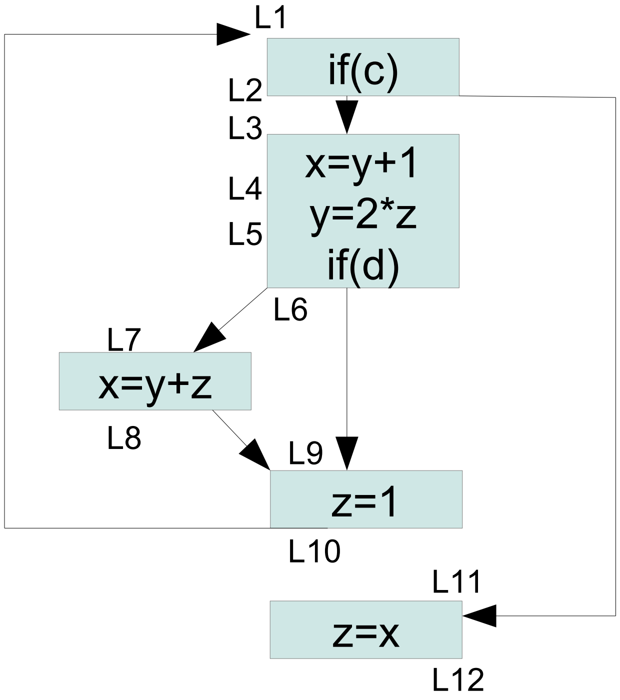
where

kill [l] = expressions “killed” by l

gen [l] = expressions “generated” by l

x = y OP z	generates {y OP z}, kills expr. with x in them
x = OP y	generates {OP y}, kills expr. with x
x = y	generates nothing, kills expr. with x
x = & y	generates nothing, kills expr. with x
if (x)	generates nothing, kills nothing
return x	generates nothing, kills nothing
x = f (y1, y2, …, yn)	generates nothing, kills expr with x

Expressions in the example



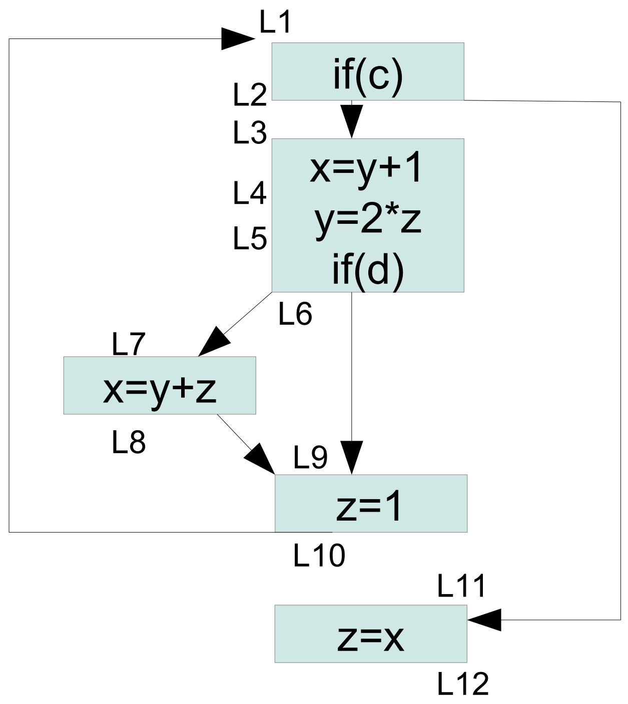
e1: y + 1

e2: 2 * z

e3: y + z

e1: y +1
e2: 2 * z
e3: y + z

Data flow equations



$L4 = \{ L3 \} \cup \{e1\}$

$L5 = \{ L4 - e1 \} \cup e2$

$L8 = \{ L7 \} \cup e3$

$L9 = L6 \cap L8$

$L10 = L9 - \{e2,e3\}$

$L12 = L11 - \{e2,e3\}$

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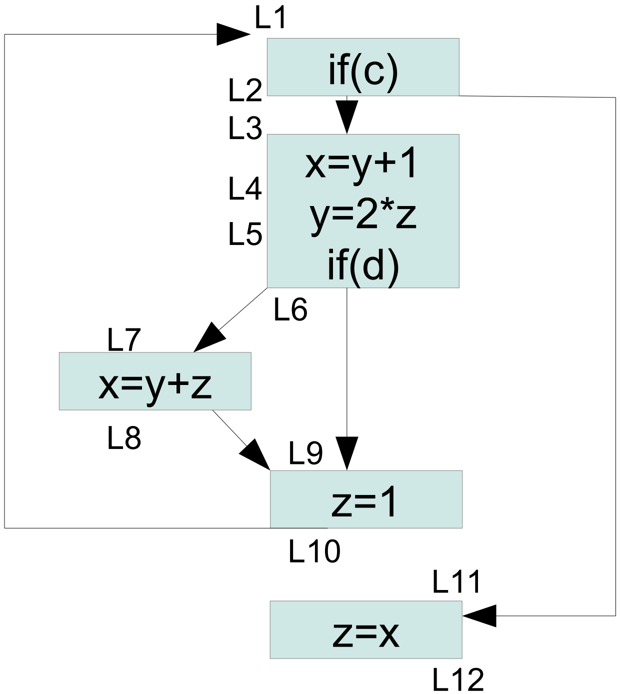
e1: y +1
e2: 2 * z
e3: y + z

L1 = {} ⋂ L10
L4 = { L3 } ⋃ {e1}
L5 = { L4 – e1 } ⋃ e2
L8 = { L7 } ⋃ e3
L9 = L6 ⋂ L8
L10 = L9 – {e2,e3}
L11 = L1
L12 = L11– {e2,e3}

Iteration 1



L1 =

L2 =

L3 =

L4 = e1

L5 = {e1 – e1 } ⋃ e2 = e2

L6 = e2

L7 = e2

L8 = e2,e3

L9 = {e2} ⋂ {e2,e3} = e2

L10 = {e2 – e2} = {}

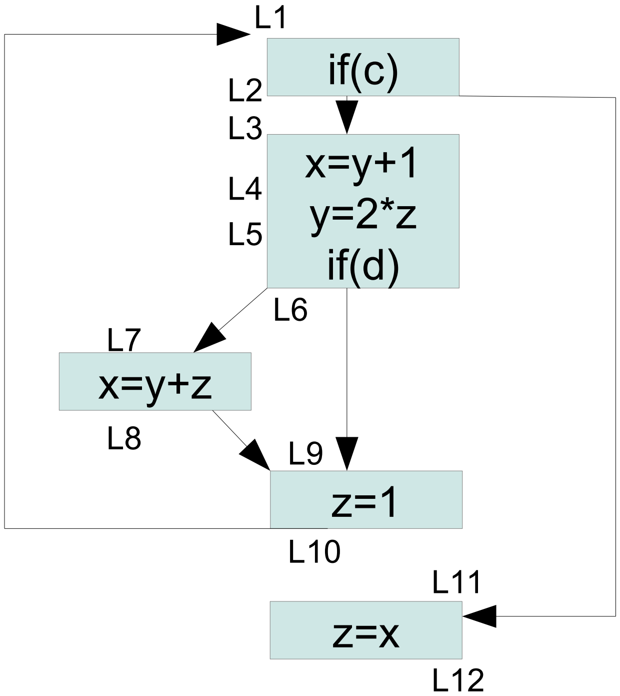
L11 =

L12 =

Iteration 2: no change

e1: y +1
e2: 2 * z
e3: y + z

L1 = {} ⋂ L10
L4 = { L3 } ⋃ {e1}
L5 = { L4 – e1 } ⋃ e2
L8 = { L7 } ⋃ e3
L9 = L6 ⋂ L8
L10 = L9 – {e2,e3}
L11 = L1
L12 = L11– {e2,e3}



L1 =
L2 =
L3 =
L4 = e1
L5 = e2
L6 = e2
L7 = e2
L8 = e2,e3
L9 = e2
L10 =
L11 =
L12 =

Reaching Definitions

- A *reaching definition* is a definition of a variable where the assigned value *may* be observed at a program point in some execution
- The sets we look for are sets of assignments, so we need to number all of those

Reaching Definitions
Data flow equations

- Instructions:

  $out[B] = F_B(in[B])$ (Forward analysis)

- Control flow:

  $in[B] = \bigcup \{ out[B'] \mid B' \in pred(B) \}$ (Meet op. is union)

- Interpretation:

  A definition reaches the entry of block B if it reaches the exit of at least one of its predecessor nodes

Reaching Definitions Transfer function

$F_I(X) = \{ X - \text{kill}[I] \} \cup \text{gen}[I]$

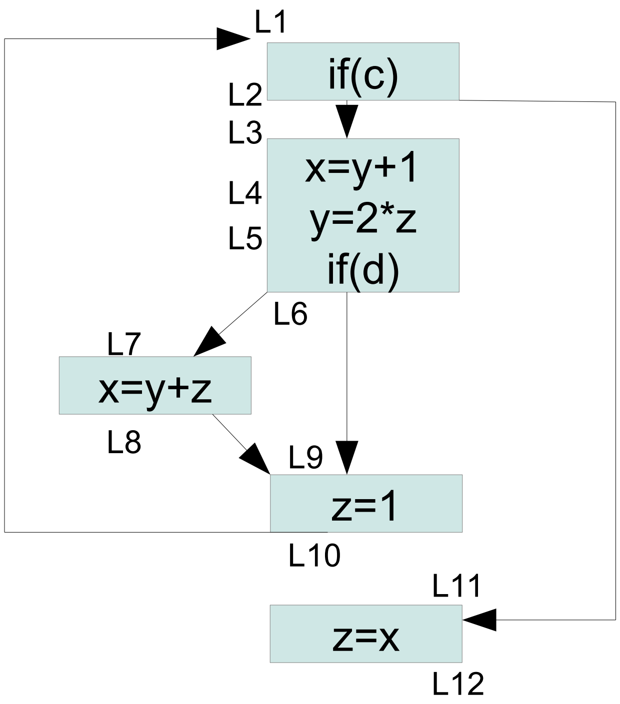
where

kill [I] = definitions “killed” by I

gen [I] = definitions “generated” by I

x = y OP z	generates {x = y OP z}, kills other definitions of x
x = OP y	generates {x = OP y}, kills other definitions of x
x = y	generates {x = y}, kills other definitions of x
x = & y	generates {x = &y}, kills other definitions of x
if (x)	generates nothing, kills nothing
return x	generates nothing, kills nothing
x = f (y1, y2, …, yn)	generates {x = f…}, kills other definitions of x

Definitions in the example



d1: x = y +1

d2: y = 2 * z

d3: x = y + z

d4: z = 1

d5: z = x

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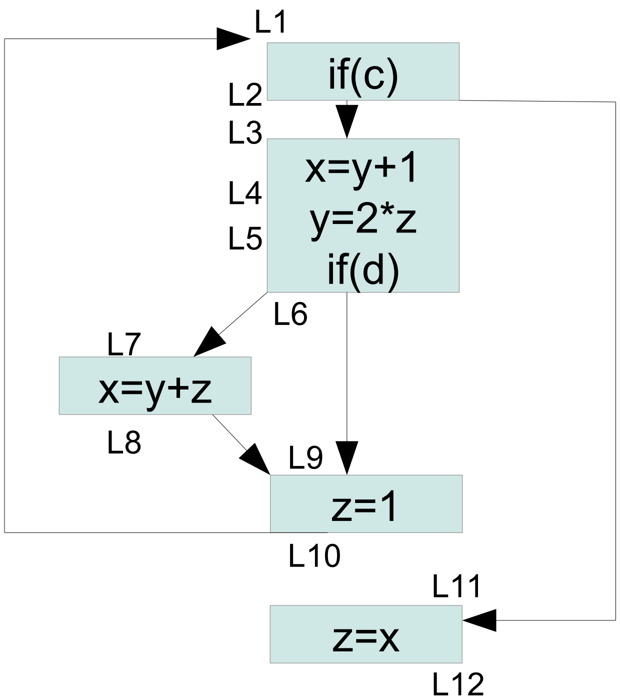
Data flow equations

d1: x = y+1
d2: y = 2*z
d3: x = y+z
d4: z = 1
d5: z = x



$L1 = \{\} \cup L10$

$L4 = \{L3-d3\} \cup d1$

$L5 = L4 \cup d2$

$L8 = \{L7-d1\} \cup d3$

$L9 = L6 \cup L8$

$L10 = \{L9-d5\} \cup d4$

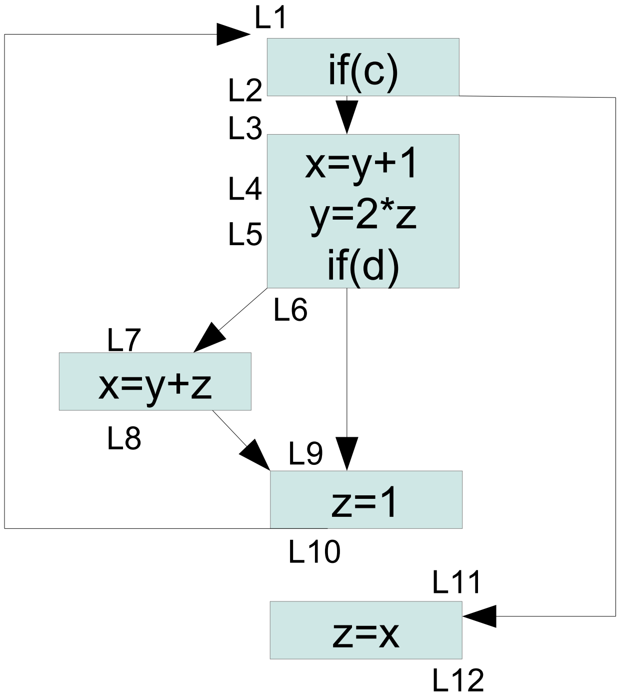
$L11 = L2$

$L12 = \{L11-d4\} \cup d5$

Iteration 1

d1: x = y+1
d2: y = 2*z
d3: x = y+z
d4: z = 1
d5: z = x

L1 = {} ⋃ L10
L4 = {L3-d3} ⋃ d1
L5 = L4 ⋃ d2
L8 = {L7-d1} ⋃ d3
L9 = L6 ⋃ L8
L10 = {L9-d5} ⋃ d4
L11 = L2
L12 = {L11-d4} ⋃ d5



L1 =

L2 =

L3 =

L4 = d1

L5 = d1,d2

L6 = d1,d2

L7 = d1,d2

L8 = d2,d3

L9 = {d1,d2} ⋃ {d2,d3} = {d1,d2,d3}

L10 = {d1,d2,d3} ⋃ d4 = {d1,d2,d3,d4}

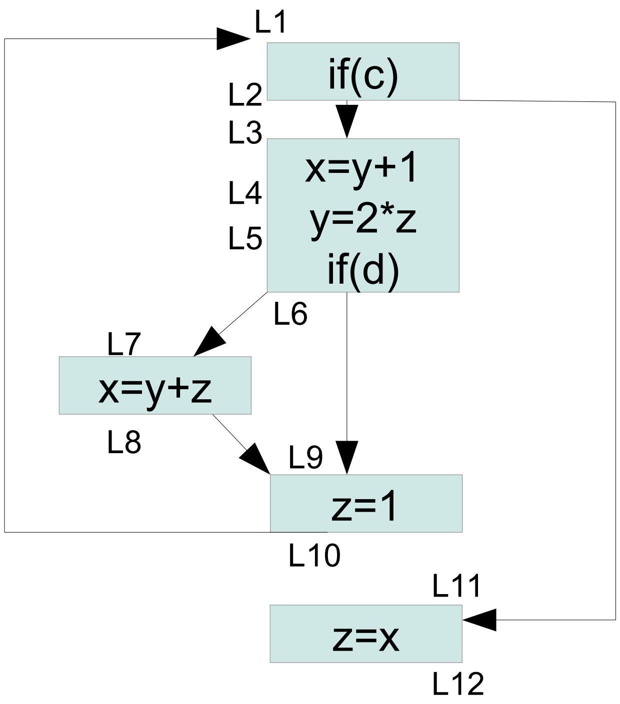
L11 =

L12 = d5

Iteration 2

d1: x = y+1
d2: y = 2*z
d3: x = y+z
d4: z = 1
d5: z = x

L1 = {} ⋃ L10
L4 = {L3-d3} ⋃ d1
L5 = L4 ⋃ d2
L8 = {L7-d1} ⋃ d3
L9 = L6 ⋃ L8
L10 = {L9-d5} ⋃ d4
L11 = L2
L12 = {L11-d4} ⋃ d5



L1 = **{} ⋃ {d1,d2,d3,d4}**

L2 = **d1,d2,d3,d4**

L3 = **d1,d2,d3,d4**

L4 = d1,**d2,d4**

L5 = d1,d2,**d4**

L6 = d1,d2,**d4**

L7 = d1,d2,**d4**

L8 = d2,d3,**d4**

L9 = d1,d2,d3 ⋃ **{d1,d2,d4} = {d1,d2,d3,d4}**

L10 = d1,d2,d3,d4

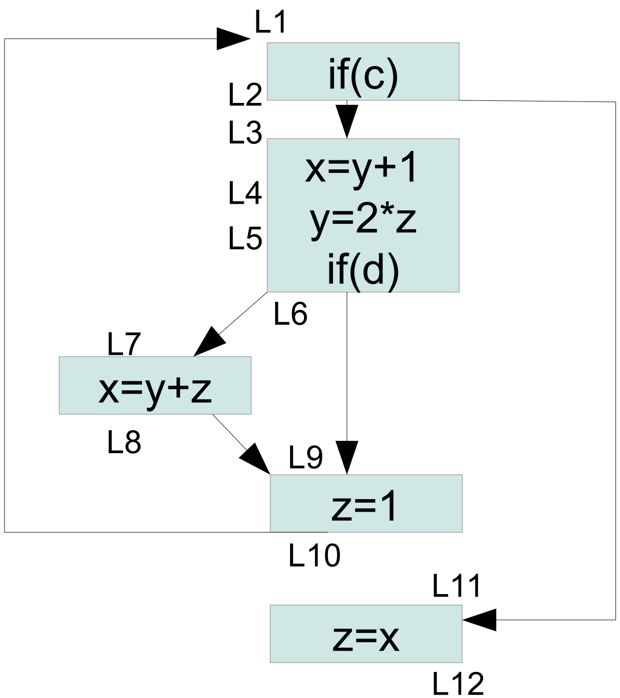
L11 = **d1,d2,d3,d4**

L12 = d5 ⋃ **d1,d2,d3 = {d1,d2,d3,d5}**

Iteration 3: no change

d1: x = y+1
d2: y = 2*z
d3: x = y+z
d4: z = 1
d5: z = x

L1 = {} ⋃ L10
L4 = {L3-d3} ⋃ d1
L5 = L4 ⋃ d2
L8 = {L7-d1} ⋃ d3
L9 = L6 ⋃ L8
L10 = {L9-d5} ⋃ d4
L11 = L2
L12 = {L11-d4} ⋃ d5



L1 = d1,d2,d3,d4
L2 = d1,d2,d3,d4
L3 = d1,d2,d3,d4
L4 = d1,d2,d4
L5 = d1,d2,d4
L6 = d1,d2,d4
L7 = d1,d2,d4
L8 = d2,d3,d4
L9 = d1,d2,d3,d4
L10 = d1,d2,d3,d4
L11 = d1,d2,d3,d4
L12 =d1,d2,d3,d5

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Key takeaways

- The choice of domain determines what we're analyzing
- With union as meet operator, we get "maybe"-analyses
  - There is a path where an element was introduced
- With intersection as meet operator, we get "must"-analyses
  - Every path introduces these elements

Next time

- We will
  - put all these (and one more) into a big ol' overview
  - take out the lattices again, and try to say something about how well the fixed point solution characterizes the analyzed program
  - invent a function which lets us use the same method to detect loops