



**NTNU – Trondheim**  
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## Type checking

# Where we left off

- We have introduced inference rules
  - And connected them to syntax tree traversal
- We have talked about instantiating inference rules for a simple ternary expression
  - And how it relates to type checking
- We'll continue now with
  - Rules for type checking some different types of statements
  - Connection to syntax tree traversal
  - Static vs. dynamic type checking



# Axioms

- Some statements don't need any premises in order to determine their type

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$\text{env} \vdash \text{true} : \text{bool}$

reads that “true” is a boolean value in any environment,  
similarly,

---

$\text{env} \vdash 42 : \text{int}$

doesn't depend on the environment either



# Declarations

- These affect the environment, that's what they're for

$$\frac{\text{env} \vdash E : T \quad \text{env} [\text{id} : T] \vdash (S2 ; S3 ; \dots ; S_n) : T'}{\text{env} \vdash \text{id} : T = E ; (S2 ; S3 ; \dots ; S_n) : T'}$$

# Assignments

- Identifiers

$$\frac{\text{env } [id : T] \vdash E : T}{\text{env } [id : T] \vdash id = E : T}$$

- Arrays

$$\frac{\text{env } \vdash E1 : \text{array}(T) \quad \text{env } \vdash E2 : \text{int} \quad \text{env } \vdash E3 : T}{\text{env } \vdash E1[E2] = E3 : T}$$

# An abbreviation

- There is, implicitly, always an environment containing the context of the statement
- We don't always need to refer to any part of it, so

$$\frac{\text{env} \vdash E1 : \text{array}(T) \quad \text{env} \vdash E2 : \text{int} \quad \text{env} \vdash E3 : T}{\text{env} \vdash E1[E2] = E3 : T}$$

might as well be written

$$\frac{E1 : \text{array}(T) \quad E2 : \text{int} \quad E3 : T}{E1[E2] = E3 : T}$$

without loss of information.

- When there *is* something to say about the env. contents,

$$\frac{}{\text{env} [id : T] \vdash E : T}$$

$$\text{env} [id : T] \vdash id = E : T$$

might as well just highlight the part we need, *i.e.*

$$\frac{id : T \vdash E : T}{id : T \vdash id = E : T}$$



# Expressions

- We looked a little bit at these already;

$$\frac{E1 : \text{int} \quad E2 : \text{int}}{E1 + E2 : \text{int}}$$

$E1 + E2 : \text{int}$

specifies that a sum of ints is an int,

$$\frac{E1 : \text{int} \quad E2 : \text{long}}{E1 + E2 : \text{long}}$$

$E1 + E2 : \text{long}$

suggests that adding promotes int to long

(or we could write

$$\frac{E1 : T1 \quad E2 : T2}{E1 + E2 : \text{lub}(T1, T2)}$$

$E1 + E2 : \text{lub}(T1, T2)$

← (“lub” = “least upper bound”)

and specify a partial order of types...)



# Whiles and sequences

E : bool            S : T

while(E) S : void

S1 : T1                            S2; S3; S4; ...; Sn : T'

S1; S2; S3; S4; ...; Sn : T'



# Function calls

- The type of a function can be written as the (Cartesian) product of its argument types, and its return type:

$$T_1 \times T_2 \times T_3 \times \dots \times T_n \rightarrow Tr$$

- Syntax-wise, calls are a case of expressions

$$\frac{E : T_1 \times T_2 \times T_3 \times \dots \times T_n \rightarrow Tr \quad E_1:T_1 \quad E_2:T_2 \quad \dots}{E ( E_1, E_2, E_3, \dots, E_n ) : Tr}$$



# Function declarations

- Suppose a declaration consists of a return type and a name,  
     $\text{Tr id}$   
a list of parameters,  
     $(T1\ p1, T2\ p2, \dots, Tn\ pn)$   
and a body which evaluates to something,  
     $\{ E; \}$   
for a grand total of  
     $\text{Tr id} ( T1\ p1, T2\ p2, \dots, Tn\ pn ) \{ E; \}$
- What we want is to check  $E$  in an environment where all the parameters have their declared types, so put them in there, and expect  $E$  to check out as the return type

# Function declarations

$$\frac{p1:T1, p2:T2, \dots, pn:Tn \vdash E : Tr}{\vdash Tr \text{ id } ( T1 \ p1, T2 \ p2, \dots, Tn \ pn ) \{ E; \} : \text{void}}$$

- Somewhere inside  $E$ , a return statement must resolve to the return type  $Tr$ 
  - How to check it? Return values don't appear in the local environment of the function...

# Return statements

- Use a placeholder in the environment
- If we introduce a “magic” variable `ret` with the return type

$$\frac{p1:T1, p2:T2, \dots, pn:Tn, \text{ret} : Tr \vdash E : Tr}{\vdash Tr \text{ id } ( T1 p1, T2 p2, \dots, Tn pn ) \{ E; \} : \text{void}}$$

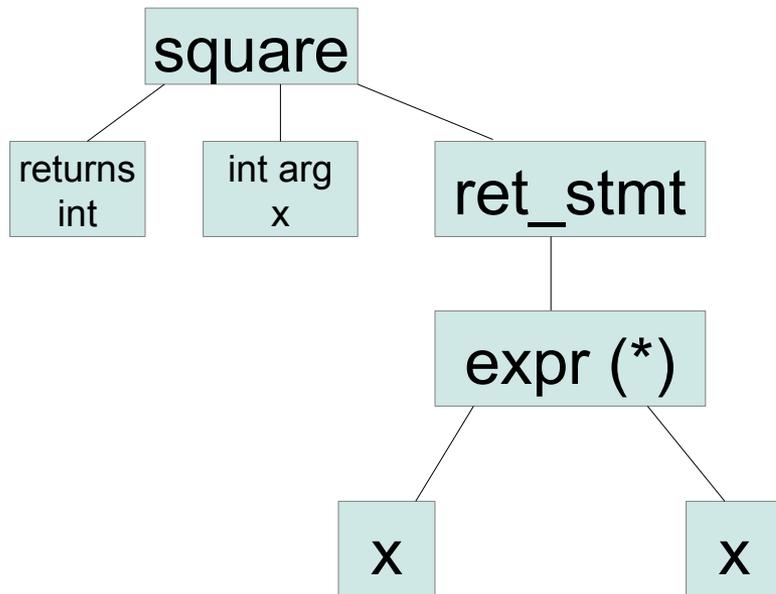
return statements can be checked as

$$\frac{\text{ret} : T \vdash E : T}{\text{ret} : T \vdash \text{return } E : \text{void}}$$


# What a type-check must do

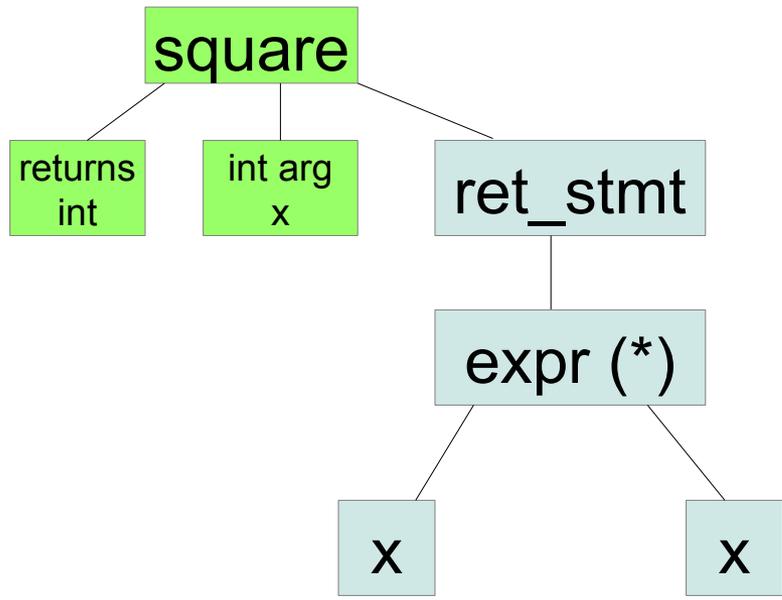
Let's define a function:

```
int square ( int x ) { return (x*x); }
```



# What a type-check must do

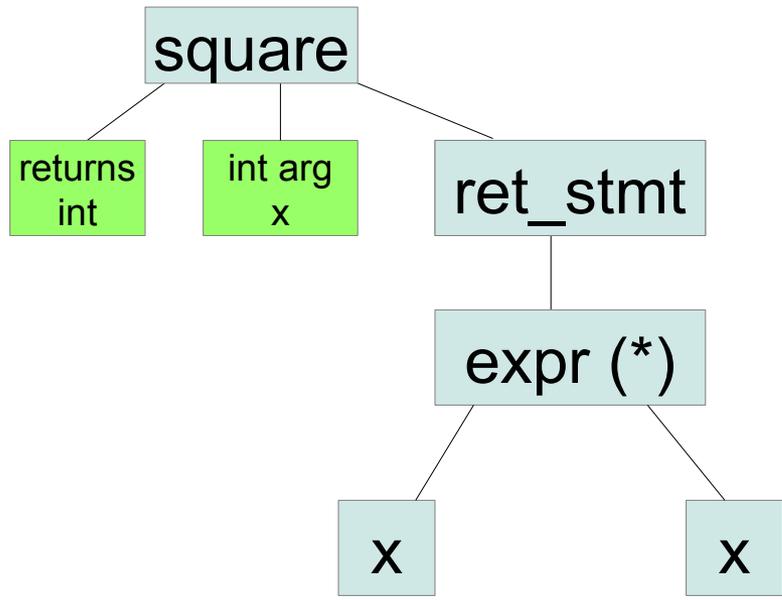
Enter the function in a global symbol table



<b>Global symbols</b>		
Name	Type	...
Square	function, int → int	

# What a type-check must do

Create a local context (either in the global table, or make another)

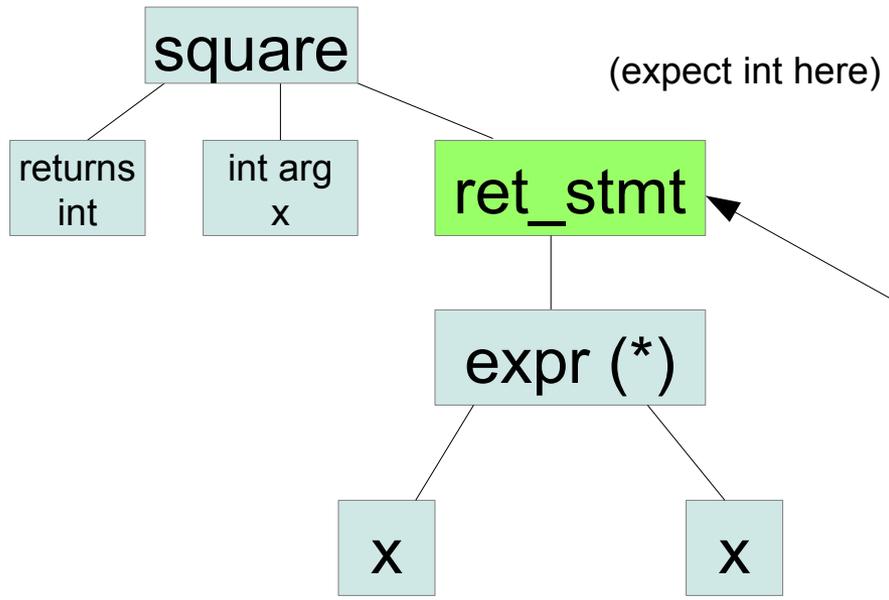


<b>Global symbols</b>		
Name	Type	...
Square	function, int → int	

<b>Square's symbols</b>		
Name	Type	...
x	int	
ret	int	

# What a type-check must do

Check statements in the function body



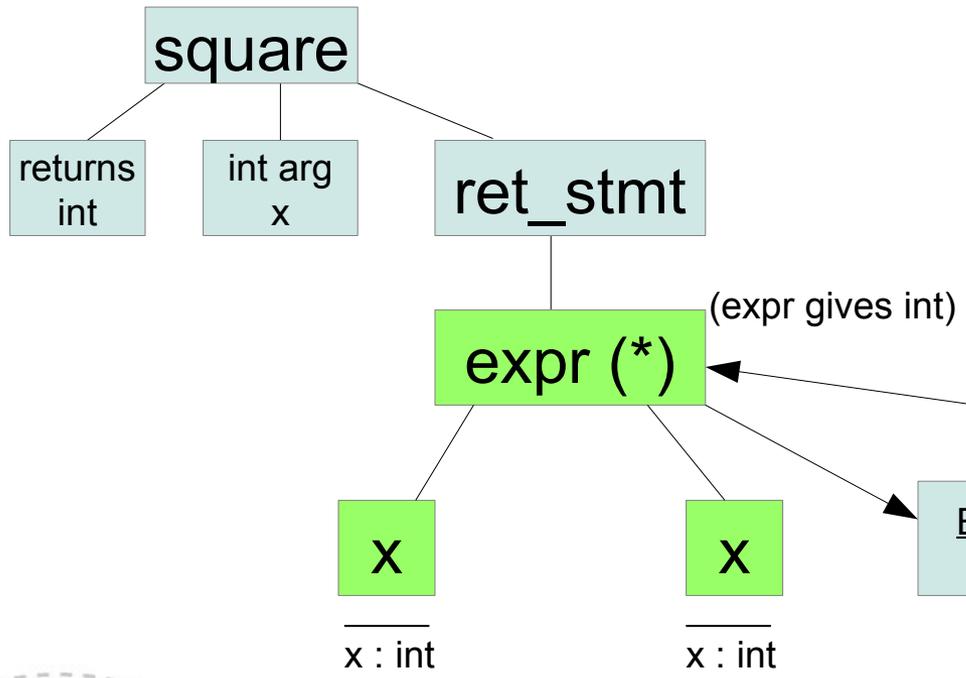
<b>Global symbols</b>		
Name	Type	...
Square	function, int → int	...

<b>Square's symbols</b>		
Name	Type	...
x	int	...
ret	int	...

$$\frac{\text{ret} : \text{int} \quad E : T}{\text{ret} : \text{int} \mid\text{- return } E : \text{void}}$$

# What a type-check must do

Check each part of each statement



<b>Global symbols</b>		
Name	Type	...
Square	function, int → int	

<b>Square's symbols</b>		
Name	Type	...
x	int	
ret	int	

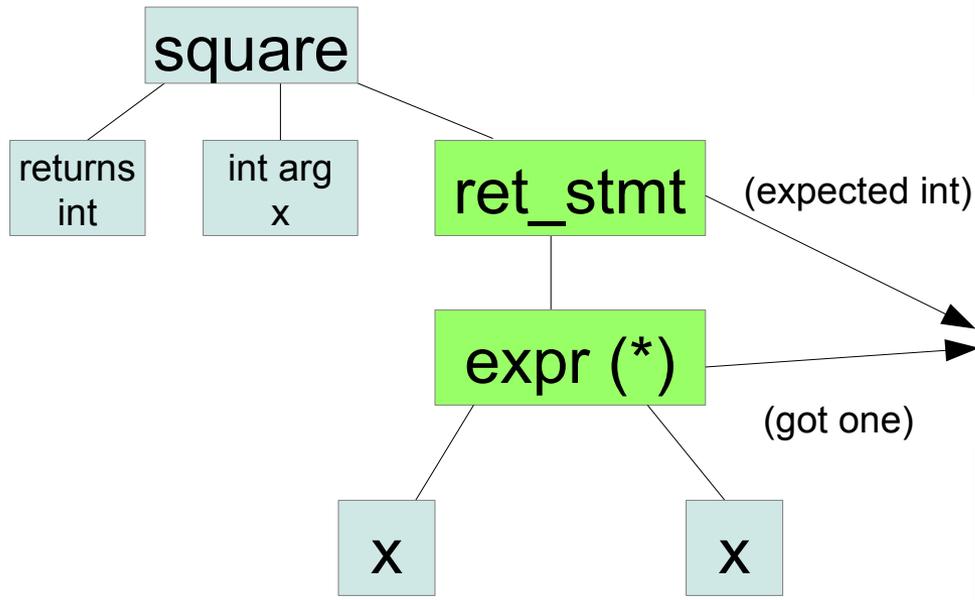
$\frac{E1 : T \quad E2 : T}{E = E1 * E2 : T}$
---

$\frac{\frac{x : int}{E1 : int} \quad \frac{x : int}{E2 : int}}{E = E1 * E2 : int}$
---

(from the table)

# What a type-check must do

Check each part of each statement



<b>Global symbols</b>		
Name	Type	...
Square	function, int → int	

<b>Square's symbols</b>		
Name	Type	...
x	int	
ret	int	

<i>(proof on prev slide)</i>	
ret : int	E : int
-----	
ret : int  - return E : void	

Hooray, 'square' is correctly typed

# Three views on checking

- Implementation-wise, we traverse the syntax tree and enforce the rules of the type system
- If the rules allow us to do that simultaneously with discovering the syntax tree, it fits a syntax-directed translation scheme *a la* Dragon
  - i.e. graft checking into the semantic actions of the parser*
- Written as inference rules, it is a construction of a proof tree which resolves a bunch of type judgments
- All the same thing, more or less

# What we've looked at is *static*

- All information about types and values comes straight from the source code
    - That's why we can do it by examining the syntax tree
    - When the compiler is finished, so is the type checking
  - It's a process of *binding*
    - Explicitly, as with “double z = 2.71828” (declaration says it)
    - Implicitly, as with “z = 3.141593” (value gives it away)
- and *checking*
- If z is consistently used as a double in the scope of this binding, the program is *type-safe*
- *Type-safety* is the lack of type errors when the program runs

# How safe is static checking?

- That depends on how it's implemented.
- C lets you lie to the type checker, under the assumption that you have control
- That includes creating type errors at run time

```
% cat square.c
double square ( double x ) { return x*x; }
% cat main.c
#include <stdio.h>
int square ( int );
int main() { printf ( "%d\n", square(64) ); }
% cc -o test main.c square.c
% ./test
4195622
% █
```

# How safe is static checking?

- Java won't have such shenanigans, and enforces more safety
- Both check statically, but according to different rules

```
% cat Square.java
public class Square { public static double square(double x) { return x*x; } }
% cat Main.java
public class Main {
    public static void main (String args[]) {
        System.out.println ( Square.square(64) );
    }
}
% javac Main.java Square.java
% java Main
4096.0
% █
```



# Dynamic types

- Other languages permit type information to appear at run time, and check it then
  - Scheme, Ruby, Python
- These are interpreted, but nothing prevents a compiler from inserting dynamic type checks into the program it generates
- Some even give you static types when you declare variables, and dynamic when you don't
  - *Dylan* pioneered this in 1995
  - C# does it today



# The strength of a type system

- Strongly typed languages guarantee that programs are type-safe if they pass checking
- Weakly typed languages admit programs that contain type errors
- A *sound* type system statically ensures that all programs are type-safe  
(Sound as in *soundness*, it doesn't make any noise)

# Strength is a design trade-off

- A program may be safe for reasons a compiler cannot detect:

```
% cat unsafe.c
#include <stdio.h>
#include <stdint.h>
int main () {
    double hello = 1.81630607015975e-310;
    puts ( (char *)&hello );
}
% make unsafe
cc      unsafe.c  -o unsafe
% ./unsafe
Hello!
% █
```

- This won't fail, but it doesn't type-check without forced casting either



# These words are not absolutes

- We saw that static checks in Java are less permissive than those in C
  - Taken as a whole, Java types also have a dynamic twist to them
  - Objects remember what type they are at run time, that's why you can get `ClassCastException`s instead of wrong answers
- Python does all its checking dynamically, and is pretty firm about consistency (stronger)

```
>>> a = 42
>>> b = "42"
>>> print a == b                                # No number is a string
False
```

- PHP also works dynamically, but has a more liberal philosophy (weaker)

```
php > $a = 42;
php > $b = "42";
php > var_dump ( $a == $b );                    # Sure, why not?
bool(true)
```

# Pros and cons of static types

## (+) *Speeeeeeeeed...*

Dynamic checking runs whenever the program does, and takes time

## (+) Evergreen analysis

- Generated result does the same thing every time it runs
- Dynamic types admit dynamic type errors

## (-) Has to be conservative

- Can't defer check until values are known, must assume they can be anything
- Stronger checking translates into accepting fewer programs



# Next up

More elaborate, derived types

- Arrays
- Records
- Objects